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PERFORMANCE ANALYSIS OF PEAK-TO -AVERAGE POWER RATIO REDUCTION TECHNIQUES IN MIMO-OFDM WIRELESS SYSTEMS

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ABSTRACT

A major drawback of Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) systems is its high Peak-to-Average Power Ratio (PAPR). Selected Mapping algorithm (SLM) and Partial Transmits Sequence (PTS) is proposed to overcome this problem. SLM is one of most a popular scheme for peak power reduction in Orthogonal Frequency Division Multiplexing (OFDM) systems by generating several OFDM symbols (candidates) carrying the same information and select the one having the lowest PAPR for transmission. In this paper we derive theoretical amplitude Probability Density Function (PDF) of the selected OFDM symbol using order statistics. This amplitude PDF enables us to derive the Signal-to-Noise Ratio (SNR) as a function of the number of candidates and error performance of MIMO-OFDM for PAPR in wireless Systems. Numerical Results show that this technique reduces the PAPR of the transmitted signals considerably.

KEYWORDS: Orthogonal Frequency Division Multiplexing (OFDM), Peak -to-Average Power Ratio (PAPR) Selective mapping (SLM), Partial Transmit sequence (PTS),

Probability Distribution Function (PDF),Signal distortion rate ( ), Bit Error Rates (BER), Signal-to-Noise Ratio (SNR).

1 INTRODUCTION

MIMO-OFDM is an attractive technique for high data rate wireless communication systems, but it exhibit a large peak-to-average power ratio (PAPR) due to the superposition of the individual signal components (the carriers) [1]. The high PAPR brings on the OFDM signal distortion in the nonlinear region of High Power Amplifier (HPA) and the signal distortion induces the degradation of Bit Error Rate (BER). In OFDM systems, several PAPR reduction schemes have been proposed to solve this problem [2-8]. One of most widely used methods is Selective Mapping (SLM) and Partial Transmit sequence (PTS) using probabilistic methods. The principle of probabilistic methods depend on reducing the probability of high PAPR by generating several OFDM symbols (multiple candidates) carrying the same information and by selecting the one having the lowest PAPR. The probabilistic method can also be classified into two strategies: sub block partitioning strategy and entire block strategy. The sub block partitioning strategy, such as partial transmits sequence (PTS) [6-8], divides frequency domain signals into several sub blocks. On the other hand, the entire block strategy, such as selected mapping (SLM) [8-10] considers the entire block for generating multiple candidates. First, the entire block strategy of the probabilistic methods to generate multiple candidates is considered and the probability Distribution Function (PDF) for the multiple candidate system is analyzed. When the candidate having the lowest PAPR is selected, the PDF of the amplitude of a selected OFDM symbol becomes a function of the number of candidates. We apply the analyzed PDF (as a function of n) to obtaining the Signal-to-Noise Ratio (SNR) as a function of n. Then, the SNR (as a function of n) can be used for analytical error performance. However, we suggest using our PDF (multiple candidates) to obtain the theoretical error performance and also the statistical channel capacity for the multiple candidate system.

2. OFDM SYSTEM MODEL

We consider the MIMO-OFDM systems with N_T transmit antennas that use N subcarriers. With OFDM modulation, a block of n data symbols (one OFDM symbol), $\{x_N, n = 0, 1, \ldots, n-1\}$ will be transmitted in parallel such that each modulates a different subcarrier from a set $\{f_N, N = 0, 1, \ldots, N-1\}$. The N subcarriers are orthogonal, i.e. $f_N = N\Delta f$ where $\Delta f = 1/nT$ and T is the symbol period. The resulting baseband OFDM signal $x_{N_T}(t)$ of a block can be expressed as

$$x_{N_T}(t) = \frac{1}{\sqrt{N}}\sum_{n=0}^{n-1} x_n e^{j2\pi f_N t} \qquad 0 \le t \le nT \tag{1}$$

Where x_n is the transmitted OFDM signal at the N^{th} subcarrier of the N_T^{th} transmit antenna. Theoretically, large peaks in OFDM system can be expressed as Peak-to-Average Power Ratio for one transmit antenna. It is usually defined as

$$\mathrm{PAPR} \triangleq \frac{\max_{0 \le t \le nT} |x_n|^2}{E\{|x_n|^2\}} = \frac{\max_{0 \le t \le nT} |x_n|^2}{\frac{1}{nT}\int_0^{nT} |x_n|^2} \tag{2}$$

Where E [·] denotes the expected value. Then, the complementary cumulative distribution function (CCDF), which is the probability that the PAPR of an OFDM symbol exceeds the given threshold PAPR0, can be expressed as

$$\mathrm{CCDF} = \Pr(\mathrm{PAPR} > \mathrm{PAPR0} = A) \qquad \text{Assume } A = \mathrm{PAPR0}. \tag{3}$$

Throughout this paper we assume the multiple candidate system and our PDF notation for several variables. n candidates (frequency domain signal) are generated by the candidate generator, where this candidate generator represents a class of probabilistic methods such as the SLM method [8–13]. After the N-point Inverse Fast Fourier Transform (IFFT), we get the n OFDM candidates (time domain signal).

If $x_{i,j} = \{x_{i,1}, x_{i,2}, \ldots, x_{i,N}\}$ $\quad i \in \{1,\ldots,n\}, \quad j \in \{1,\ldots,N\}$.

When we define $z_{i,j} \triangleq |x_{i,j}|$

Then
$$|x_1| = z_1 = \{z_{1,1}, z_{1,2}, \ldots, z_{1,N-1}, z_{1,N}\}$$
$$|x_2| = z_2 = \{z_{2,1}, z_{2,2}, \ldots, z_{2,N-1}, z_{2,N}\}$$
$$\vdots$$
$$|x_{i0}| = z_{i0} = \{z_{i0,1}, z_{i0,2}, \ldots, z_{i0,N-1}, z_{i0,N}\}$$
$$\vdots$$
$$|x_n| = z_n = \{z_{n,1}, z_{n,2}, \ldots, z_{n,N-1}, z_{n,N}\} \tag{4}$$

and the peak detector selects the $i0^{th}$ candidate, where $i0 = \arg\min_i(\max_j\{z_{i,j}\})$ for $i \in \{1,\ldots,n\}$ and $j \in \{1,\ldots,N\}$. Then, the selected ($i0^{th}$) OFDM signal candidate is clipped by a nonlinear amplifier, where we consider the soft clipping Model [14], [21] as follows:

$$\tilde{x}_{i0,j} \triangleq f(x_{i0,j}) = \begin{cases} x_{i0,j} & \text{for } |x_{i0,j}| \le A \\ A \cdot \frac{x_{i0,j}}{|x_{i0,j}|} & \text{for } |x_{i0,j}| > A \end{cases} \tag{5}$$

Where A is the maximum permissible amplitude for the clipping model (threshold level). The clipped io^{th} candidate is transmitted to the receiver with its side information, where the side information contains the information of io and it is used for recovering the original data. The side information protection depends on the various protection strategies, such as no side information method [15, 16] or coded side information method [17]. However, in this paper, for analyzing the pure effect of increasing n for the multiple candidate system; we assume that the side information is sent without errors. If we assume the complex OFDM signal, which consists of a number of independent orthogonal subcarriers and the OFDM signal $x_{i,j}$ for $i \in \{1,\dots,n\}$ and $j \in \{1,\dots,N\}$. is complex Gaussian distributed with mean 0 and variance 1, the envelope $z_{i,j} = |x_{i,j}|$ is Rayleigh distributed with PDF f_z given by:

$$f_z(z) = \begin{cases} 2z.e^{-z^2} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases} \quad (6)$$

According to the largest order statistics [15], the distribution of the maximum of the amplitude values ($max_j\{z_{i,j}\}) = f_{zmax}$ is given by

$$f_{zmax}(z) = N\, f_z(z)(\int_{-\infty}^{z} f_z(x)\,dx)^{N-1} = N\, f_z(z)(1-e^{-z^2})^{N-1} \quad (7)$$

When we select the candidate having minimum peak amplitude among n candidates, according to the smallest order statistics (CCDF) [18], we obtain the PDF of the peak amplitude of the selected candidate

$$min_i(\,max_j\{z_{i,j}\}) = \widetilde{f_{zmax}}$$

Using $f_{zmax}(z)$

$$\widetilde{f_{zmax}}(z) = n.f_{zmax}(z)\,(\int_z^{\infty} f_{zmax}(x)\,dx)^{n-1}$$

From Eq. (6), (7)

$$\widetilde{f_{zmax}}(z) = 2\,z\,n.N.e^{-z^2}\ (1-e^{-z^2})^{N-1}\ (\int_z^{\infty} f_{zmax}(x)\,dx)^{n-1}$$

Where $\int_z^{\infty} f_{zmax}(x)\,dx = 1-((1-e^{-z^2}))^N$, and $M(z) = (1-e^{-z^2})$

$$\widetilde{f_{zmax}}(z) = 2\,n\,N\,z.f_{zmax}(z).(M(z))^{N-1}.(M(z))^{N}).((1-(M(z))^N)^{n-1} \quad (8)$$

We now want to know the PDF of amplitude of the selected candidate $z_{io} = \widetilde{f_z}$ in (6), we have obtained $\widetilde{f_{zmax}}(z)$ from $f_{zmax}(z)$ using the smallest order statistics (CCDF). Furthermore, since

$$max_j\{z_{io,j}\} = min_i(\,max_j\{z_{i,j}\} = \widetilde{f_{zmax}}(z).$$

We can also express $\widetilde{f_{zmax}}(z)$ as a function of $\widetilde{f_z}(z)$ using the largest order statistics. Then,

$$\widetilde{f_{zmax}}(z) = n.\widetilde{f_z}(z)\left(\int_0^z \widetilde{f_z}(x)\,dx\right)^{N-1} = \frac{d[\widetilde{F_z}(z)^N]}{dz}$$

$$\widetilde{F_z}(z) = (\int_0^z \widetilde{f_{zmax}}(x)\,dx)^{\frac{1}{N}} \quad (9)$$

The PDF of the amplitude of the selected candidate is given by: $\widetilde{F_z}(z) = \widetilde{F_z}(z)'$

$$\widetilde{F_z}(z)' = 1/N(\int_0^z \widetilde{F_z}(x)\,dx)^{1/N-1} \cdot \widetilde{f_{zmax}}(z)$$

$$= n.f_z(z).(1-(1-(M(z))^N)^{\frac{1}{N}-1}.(M(z))^{N-1}.((1-(M(z))^N)^{n-1} \quad (10)$$

We expect all n carriers to be active. Moreover, we consider N_T transmit antennas, over which independent data streams should be communicated. Space-time coding as, e.g., in [19], [20] is not considered here. In each of the N_T parallel OFDM transmitters a block of n distinct complex-valued carriers (OFDM frame, combined into the vector is transformed into time-domain (equivalent complex-valued baseband signals) using the IFFT. Combining the time-domain samples into the vector the correspondence is written in short as usual in wireless applications. Unfortunately, because of the statistical independence of the carriers, the time-domain samples are approximately complex Gaussian distributed. This results in a high peak-to average power ratio

$$\tilde{F}_z(z) = (n.f_z(z) .(1-(1-(M(z))^N)^{\frac{1}{N}-1} .(M(z))^{N-1} .((1-(M(z))^N)^{n-1})^{N_T} \quad (11)$$

We apply (11) to obtaining the Signal-to-Noise-plus-distortion Ratio (SNR) as a function of n ($SNR^{(n)}$) [13]. Used the Rayleigh PDF f_z, to obtain the SNR of a multiple candidate system. However, the PDF of amplitude of the selected candidates not Rayleigh PDF anymore, being the function of n. Therefore, we use the PDF of (11), $\tilde{F}_z$ to obtain the SNR of multiple candidate system, and here after we will use $SNR^{(n)}$ as a function of n, instead of SNR. For that, the PAPR threshold for clipping frame error (H) is define as

$$H = (A^{(n)})^2 / P_{in}^{(n)}$$

Where the input power $p_{in}^{(n)} = \int_0^{\infty} \tilde{f}_z(x)\, dx$, and

$$p_{in}^{(n)} = E\{|x_{i,j}|^2\} = \int_0^{\infty} (z^n)^2 \tilde{f}_z(z)\, dz \quad (12)$$

From eq. (11) the total output power for the multiple candidate solution is obtained as

$$P_{out}^{(n)} = \int_0^{A^{(n)}} (z^n)^2 \tilde{f}_z(z)\, dz + \int_{A^{(n)}}^{\infty} (z^n)^2 \tilde{f}_z(z)\, dz \quad (13)$$

and the signal distortion rate, $\alpha^{(n)} = \frac{P_{out}^{(n)}}{P_{in}^{(n)}} = \frac{[\int_0^{A^{(n)}} (z^n)^2 \tilde{f}_z(z)\, dz - \int_{A^{(n)}}^{\infty} (z^n)^2 \tilde{f}_z(z)\, dz]}{P_{in}^{(n)}}$ (14)

Where the total attenuation factor $K = \frac{\varepsilon}{P_{out}^{(n)}} = \frac{(\alpha^{(n)})^2\, P_{in}^{(n)}}{P_{out}^{(n)}}$

The SNR for the multiple candidate technique is given by

$$SNR^{(n)} = \frac{K\frac{E_S}{N_O}}{1+(1-K)\frac{E_S}{N_O}} \quad (15)$$

Finally we assume that the side information is transmitted without errors, the BER modulated Signal over the AWGN channel is given by

$$P_r = Q(\sqrt{SNR^{(n)}})$$

Furthermore Symbol error rates (SER):

$$P_S = 1 - (1 - P_r)^2.$$

For the frequency-nonselective slowly (constant attenuation during one OFDM symbol) Rayleigh-fading channel

The BER is given by $P_r = \int_0^{\infty} Q\left(\sqrt{\frac{c^2 \cdot K\frac{E_S}{N_O}}{c^2 \cdot 1+(1-K)\frac{E_S}{N_O}}}\right) \tilde{f}_z(c)\, dc$ (16)

Where c is the channel attenuation which is Rayleigh distributed with $E\{|c^2|\} = 1$.

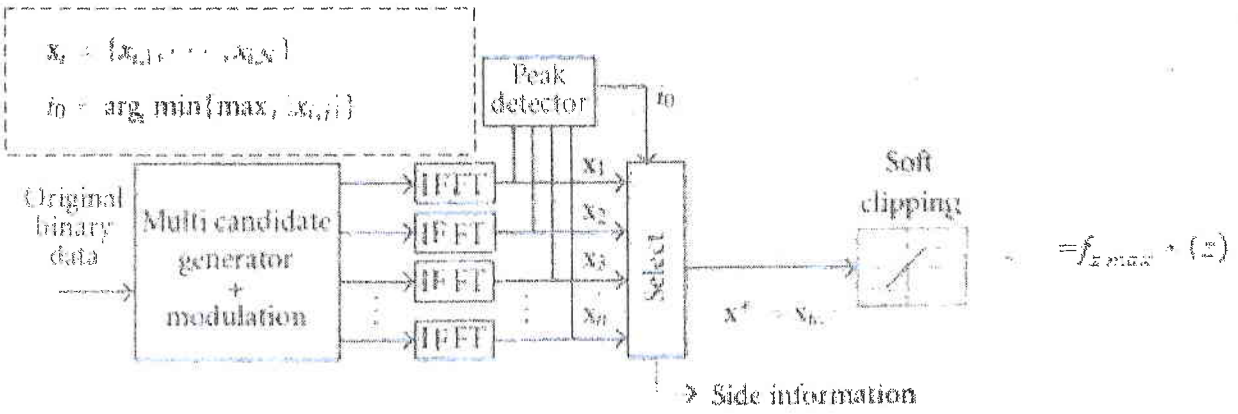

Fig. 1. a) SLM Block Diagram

3. NUMERICAL RESULTS

A SLM algorithm is non-distortion techniques for reducing PAPR in OFDM system. In order to have error-free demodulation in the receiving end, side information must be sent to the receiver, correctly. Hence, in practical application often requires the use of some coding measures to protect information from being disturbed; this paper focuses only on studying PAPR reduction performance in MIMO-OFDM system.

Figure 2. and 3. show SLM proposal can significantly improve the PAPR distributions of OFDM system, that is, significantly reduce the presenting probability of large peak power signal. The increasing of the number of OFDM signal frames n will raise the complexity dramatically, but with benefit of small improvement of PAPR reduction performance that means it can be used for different OFDM systems with different number of carriers. It is particularly suitable for the OFDM system with a large number of sub-carriers (more than 128).

Figure 4, show the PDF of amplitude of the selected candidates as the function of n where n=1, 2, 4, 8. We use the PDF of eq (10), to obtain the SNR of multiple candidate system, and here after we will use $SNR^{(n)}$ as a function n.

Figure 5. shows the error performance comparison over AWGN channel and frequency-nonselective fading channel, where the analytical Symbol Error Rate is well matched with an error frame which appears at large SNR because of the clipping noise at H = 0 dB and n= 1, 2, 4, 8. We can see that the error can decrease, by increasing n.

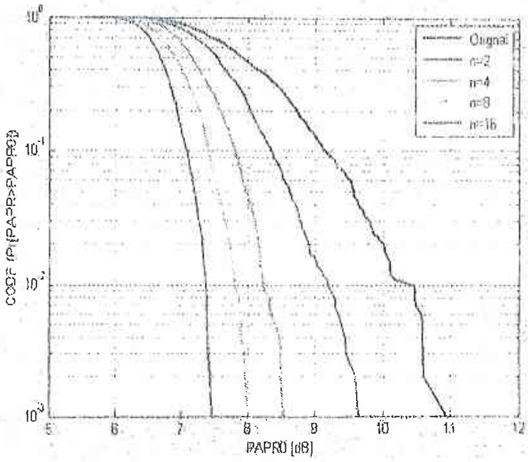

Fig.2. Comparison of PAPR reduction Performances with different values of n.

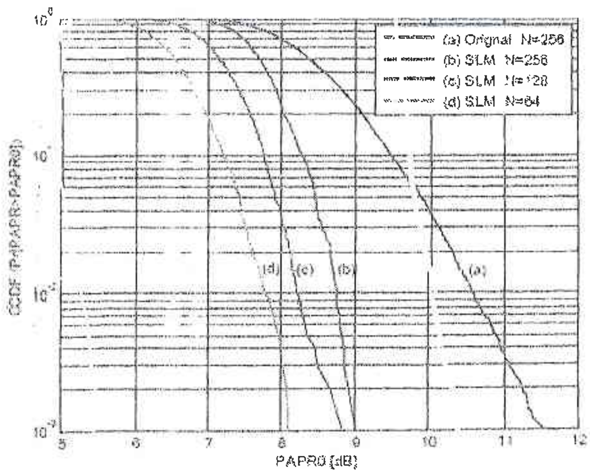

Fig.3. Comparison of PAPR reduction Performances with different values of N.

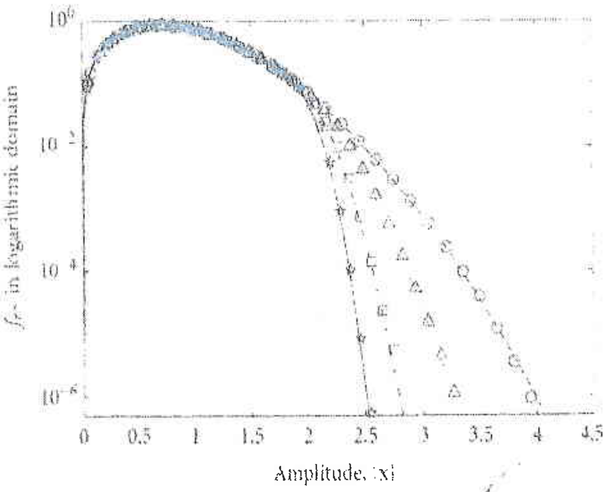

Fig. 4. Analytical PDF for $F^{(n)}_{z}(z)$ Corresponding to N = 128.

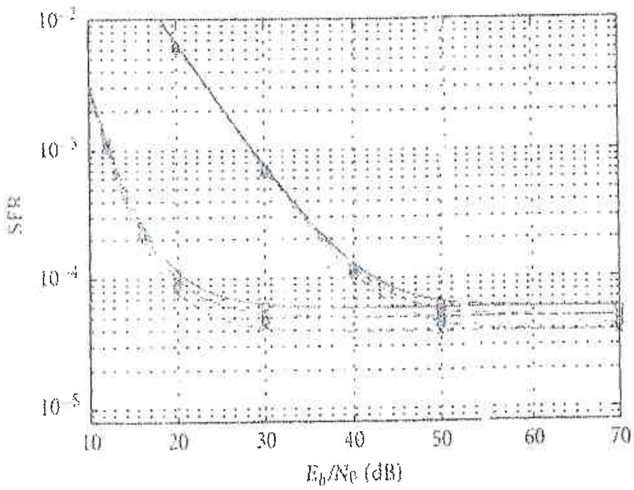

Fig. 5. Analytical Symbol Error Rate against frame Error This appears at large SNR because of the clipping Noise at H = 0 dB and n = 1, 2, 4, 8

4. CONCLUSIONS

In this paper, the performance of SLM operating jointly on blocks of OFDM frames, either in spatial or temporal direction has been assessed. Analytic expressions for the CCDF of PAPR have been derived. In contrast to other MIMO PAPR reduction schemes, using SLM the CCDF of PAPR exhibits a steeper decay, increased by a factor (almost) equal to the number of transmit antennas. We study probability density function (PDF) analysis and the signal-to-noise ratio (SINR) of a multiple candidate system for reducing the PAPR in OFDM modulation system. Since the selected OFDM symbol (candidate) has amplitude PDF which is function of the number of candidates n, the derived $SNR^{(n)}$ is also the function of n, and it can be used for estimation of theoretical error performance. In this paper, the side information is assumed not to be erroneous for analyzing the pure effect of multiple candidates. we conclude that the more the candidates, not only the better PAPR reduction performance, but also the better error performance under the assumption of side information transmission without error, and at the expense of computational complexity for n IFFT circuits.

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Dr. Sahar Abdel- Rahman
I am pleased to inform you that your manuscript entitled

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